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Three – Dimensional Generalized Characteristics of Autonomous Voltage – Fed R L C Inverters

Evgeniy Ivanov Popov, Liliya Ivanova Pindeva and Sergey Chavdarov Hristov

Abstract – **This article is aimed at obtaining and applying of the three-dimensional (surface) generalized characteristics of the basic circuits of the autonomous voltage-fed RLC inverter and its dual counterpart - the parallel current-fed inverter. The relationships between the parameters are given. The mathematical information is processed by an original MATLAB program. The characteristics are calculated, visualized and confirmed. They are convenient for circuit study and design of all inverters that can be reduced to the already mentioned ones.**

Keywords – **Three-dimensional characteristics, Voltage-fed-RLC inverters, MATLAB program**

I. Introduction

The autonomous voltage–fed R L C inverter (Fig. 1a) and its dual counterpart the autonomous current–fed R L C inverter (Fig. 1b) cover wide range of practical inverter circuits generally applied in electronic technology. In this case important mutually connected problems are the accurate design of the power circuit, appropriate adjustment between the inverter and the load, and adequate control providing stable operation of the converter when wide variations of the load are observed.

The mathematical relationships between the parameters of the inverter are rather complicated [1], [2], [3]. It can be proved that all the quantities in these second order topologies depend on two variables - the ratio between the controlling angular frequency and the generalized angular frequency ω/Ω on one hand, and the ratio between the damping coefficient and the generalized angular frequency δ/Ω of the power circuit, on the other. At the same time engineering practice in the area of the power electronics would like to have fast and accurate means for simultaneous solution of the stated problems. Such means would be the characteristics of the circuits under investigation. The generalized characteristics of the already mentioned semiconductor R L C inverters, which are obtained and graphically displayed in [2] (older variants are given in [4], [5]) are two – dimensional. This fact makes them not very accurate and convenient to use. The mutual influence between the parameters of the inverter is not very easily gathered.

The aim of this article is to obtain practically applicable and accurate three – dimensional generalized characteristics of the autonomous voltage – fed R L C inverter and the related to it current – fed inverter, which show in a convenient way the mutual dependence of inverter parameters. To fulfill this aim a rather sophisticated original MATLAB program has been created and run.

E. Popov is with the Department of Power Electronics, Faculty of Electronic Engineering and Technologies, Technical University - Sofia, 8 Kliment Ohridski blvd., 1000 Sofia, Bulgaria, e-mail: epopov@tu-sofia.bg

L. Pindeva is with the Department of Chemistry, Faculty of Electronic Engineering and Technologies, Technical University - Sofia, 8 Kliment Ohridski blvd., 1000 Sofia, Bulgaria,

S. Hristov is with the Department of Power Electronics, Faculty of Electronic Engineering and Technologies, Technical University - Sofia, 8 Kliment Ohridski blvd., 1000 Sofia, Bulgaria, e-mail: sergo_zmeia@yahoo.com

II. Problem Approach

The relationships between the inverter (Fig. 1a) parameters for continuous load current mode and different character of RLC circuit can be expressed in a common manner as follows.

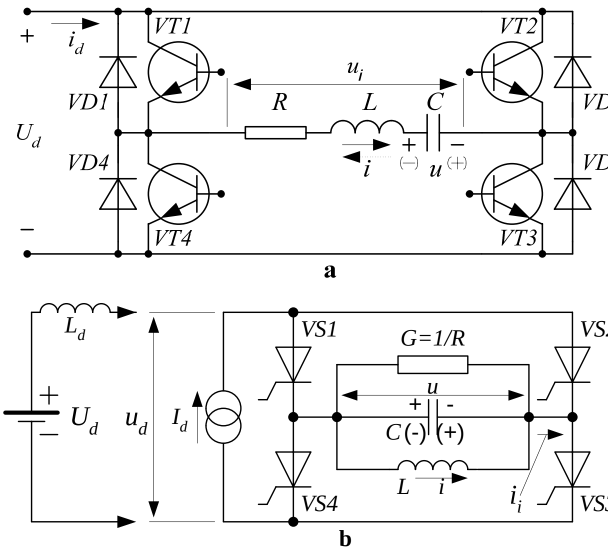

Fig. 1. Autonomous voltage – fed RLC inverter (a) and current – fed RLC inverter (b).

The damping coefficient is

(1) $$\delta = R/(2L)$$

For oscillatory mode $R < 2\sqrt{L/C}$ the generalized frequency (the inherent frequency of the serial inverter circuit) is

(2) $$\Omega = \omega_0 = \sqrt{1/(LC) - \delta^2}$$

and the following designations are used $c = +1$, $f_S(x) = \sin x$, $f_C(x) = \cos x$.

For over damped mode $R > 2\sqrt{L/C}$ the generalized frequency is

(3) $$\Omega = \sqrt{\delta^2 - 1/(LC)}$$

and the designations are $c = -1$, $f_S(x) = \sinh x$, $f_C(x) = \cosh x$. For critical mode $R = 2\sqrt{L/C}$ the generalized frequency is

(4) $$\Omega = \delta$$

and the designations are $c = 0$, $f_S(x) = x$, $f_C(x) = 1$.

There are several systems of parameters for description of the behavior of the inverter circuit. Their explanations, comments and mutual relationships are given in [2] in details. The system that covers all modes of operation includes the load coefficient B and the power factor $\cos\varphi$ [4], [5]. For Fig. 1a they are

(5) $$B = \frac{\sqrt{1+\omega^2C^2R^2}}{\omega^2LC} = \frac{R}{\omega L\cos\varphi},$$

(6) $$\cos\varphi = \frac{\omega CR}{\sqrt{1+\omega^2C^2R^2}}$$

For Fig. 1b (5) and (6) are valid after exchanging $C \to L$, $L \to C$, $R \to G = 1/R$. For the other parameters the exchange also includes $u \to i$, $i \to u$, etc..

The ratio between the controlling angular frequency and the generalized angular frequency for oscillatory or over damped modes is

(7) $$n_\omega = \frac{\omega}{\Omega} = \frac{2}{\sqrt{c(4B\sin\varphi - B^2\cos^2\varphi)}}$$

and for critical mode it is

(8) $$n_\omega = \frac{\omega}{\Omega} = \frac{\omega}{\delta} = \frac{2}{B\cos\varphi}$$

The ratio between the damping coefficient and the generalized angular frequency for oscillatory or over damped modes is

(9) $$n_\delta = \frac{\delta}{\Omega} = \frac{B\cos\varphi}{\sqrt{c(4B\sin\varphi - B^2\cos^2\varphi)}}$$

and for critical mode it is

(10) $$n_\delta = \frac{\delta}{\Omega} = \frac{\delta}{\delta} = 1$$

The main parameters of the inverter circuit (Fig. 1a) are determined as follows [2]. The angle

(11) $$\theta_2 = \pi / n_\omega$$

The parameter a and the angle θ_1 are determined from

(12) $$a = \frac{f_S(\theta_1)/f_C(\theta_1)}{1 - n_\delta . f_S(\theta_1)/f_C(\theta_1)} = \frac{f_S(\theta_2)}{e^{n_\delta.\theta_2} + f_C(\theta_2) - n_\delta . f_S(\theta_2)}$$

The generalized coefficient of hesitation is

(13) $$K = \frac{1}{1 + e^{-n_\delta.\theta_2}[(ca + n_\delta + an_\delta^2) f_S(\theta_2) + f_C(\theta_2)]}$$

The average value of the input current (all values are normalized) is

(14) $$I'_d = \frac{I_d \Omega L}{U_d} = \frac{1}{\theta_2}\int_0^{\theta_2} i'(\theta)d\theta = \frac{1}{\theta_2}.\frac{2(2K-1)}{n_\delta^2 + c}$$

The RMS value of the inverter current is

(15) $$I' = \frac{I\Omega L}{U_d} = \sqrt{I'_d/(2.n_\delta)}$$

The *output characteristic* is

(16) $$OCH = I'/I'_d$$

The *input characteristic* is

(17) $$ICH = 1/(n_\omega.I'_d)$$

The *characteristic of the coefficient of nonlinear distortion* (klir – factor) of the inverter current is

(18) $$kf[\%] = 100.\sqrt{I'^2 - I'^2_{(1)}}\ / I'_{(1)},$$

where the m-th harmonic component of the inverter current ($m = 1,3,5,7,...$) is

(19) $$I'_{(m)} = \sqrt{8}/\{m\pi\sqrt{4n_\delta^2 + [m.n_\omega - (c + n_\delta^2)/m/n_\omega]^2}\}$$

The stated relationships are valid for continuous inverter current without interruptions which is the most common case. For discontinuous inverter current with interruptions they can be derived from the already given expressions and most of them can be found in [3]. The different expressions for this case are as follows: $\theta_2 = 2\pi$, $\theta_1 = \pi$, $a = 0$. The generalized coefficient of hesitation is

(20) $$K = \frac{1}{1 + e^{-2.\pi n_\delta.}}$$

The input current is

(21) $$I'_d = \frac{I_d\Omega L}{U_d} = \frac{n_\omega^2}{\pi}.\frac{2(2K-1)}{n_\delta^2 + c}$$

The m-th harmonic component of the inverter current ($m = 1,3,5,7,...$) for discontinuous inverter current mode is

(22) $$I'_{(m)} = \sqrt{8}/\{m\pi\sqrt{4n_\delta^2 + [mn_\omega - (c + n_\delta^2)/m/n_\omega]^2}\}. .\cos(m\pi/2 - mn_\omega\pi)$$

For Fig. 1a the reactive power of the capacitor C is

(23) $$P'_{Cr} = P_{Cr}/(U_d I_d) = (n_\delta^2 + c)/n_\omega / I'_d . \sum_{m=1,3,5,...}^{\infty} I'^2_{(m)}/m$$

For the same circuit the reactive power of the inductance L is

(24 $$P'_{Lr} = P_{Lr}/(U_d I_d) = n_\omega / I'_d . \sum_{m=1,3,5,...}^{\infty} I'^2_{(m)}/m$$

For Fig. 1b the corresponding parameters can be easily found taking into consideration its duality to Fig. 1a.

The *turn – off (recovery) characteristic* has a physical meaning only for thyristor inverters. For thyristor resonant inverter with free – wheeling diodes having a capacitive reaction of its diagonal RLC circuit $n_\omega < 1$ (Fig. 1a) this characteristic is expressed by

(25) $$TQC = (\theta_2 - \theta_1)[rad].n_\omega/(2.\pi)$$

For discontinuous inverter current mode ($n_\omega < 0.5$)

(26) $$TQC = n_\omega / 2$$

For thyristor current – fed inverter (Fig. 1b), operating in over damped, critical or oscillatory mode with capacitive reaction of the diagonal RLC circuit, the recovery characteristic is given by

(27) $$TQC = \theta_1[rad].n_\omega/(2.\pi)$$

These are the most important parameters and related to them characteristics. All other parameters and characteristics can be very easily determined and calculated [2].

It can be concluded that all parameters are functions of n_ω, n_δ and consequently of the couple of main parameters. (B, $\cos\varphi$ in this case). For any other couple of main parameters it is necessary to take into account their description according to [2].

III. Processing of Information

An original MATLAB program has been created for processing the whole mathematical information, calculation of the inverter parameters, generation of the generalized characteristics and their visualization in three – dimensional form as surface characteristics. It should be underlined the difficulty of the program specifics in respect to the preparation of the array containing the digital information corresponding to a given characteristic. Anyone who would like to have this MATLAB program is kindly advised to contact the first author of the present paper via e-mail.

IV. 3-D Generalized Characteristics

The mode of inverter operation could be easily seen from Fig. 2.

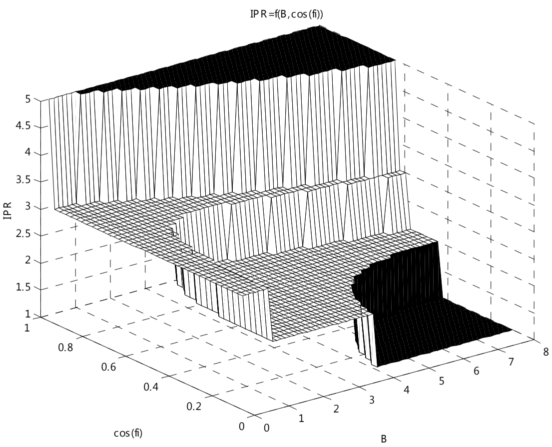

Fig. 2. The mode of inverter operation depending on $B, \cos\varphi$.

The output characteristic $OCH = f(B, \cos\varphi)$ is graphically displayed in Fig. 3.

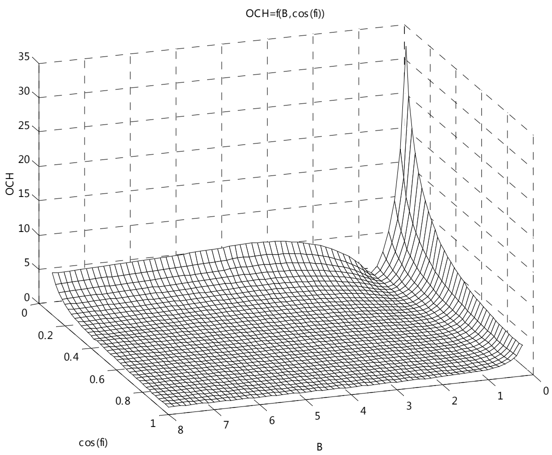

Fig. 3. The 3-D output characteristic $OCH = f(B, \cos\varphi)$

The input characteristic $ICH = f(B, \cos\varphi)$ is graphically displayed in Fig. 4.

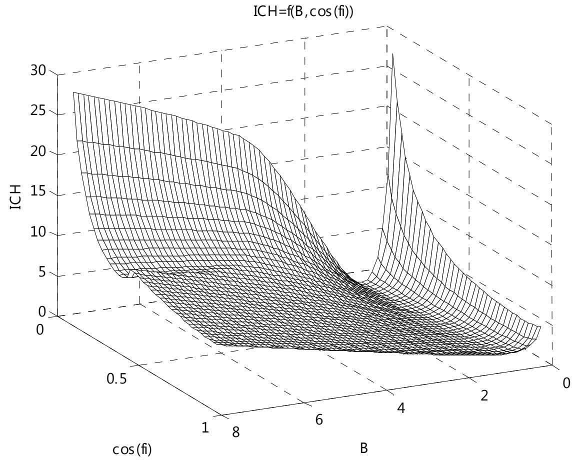

Fig. 4. The 3-D input characteristic $ICH = f(B, \cos\varphi)$

The characteristic of the coefficient of nonlinear distortion of the inverter (Fig. 1a) current $kf[\%] = f(B, \cos\varphi)$ is graphically displayed in Fig. 5.

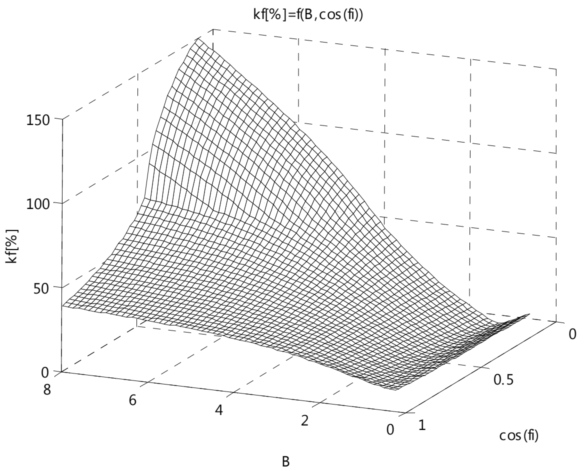

Fig. 5. The 3-D characteristic of the coefficient of nonlinear distortion $kf[\%] = f(B, \cos\varphi)$

The turn – off (recovery) characteristic $TQC = f(B, \cos\varphi)$ is graphically displayed in Fig. 6.

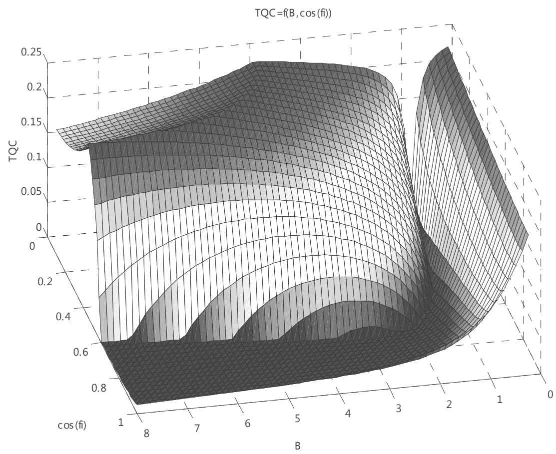

Fig. 6. The 3-D turn – off (recovery) characteristic $TQC = f(B, \cos\varphi)$

The characteristic of the normalized reactive power of the capacitor C in Fig. 1a $P'_{Cr} = P_{Cr} / P_d = f(B, \cos\varphi)$ is graphically displayed in Fig. 7.

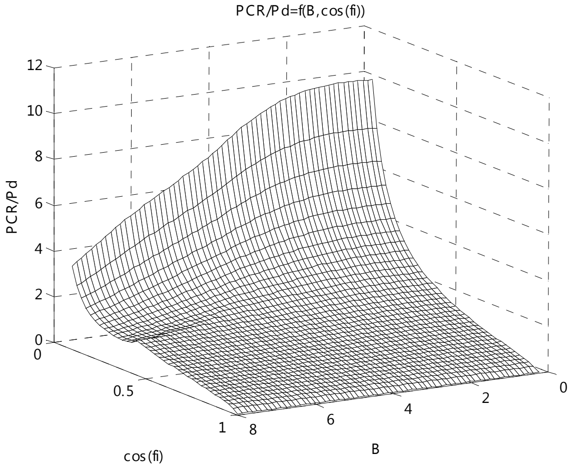

Fig. 7. The 3-D characteristic of the normalized reactive power of the capacitor C $P'_{Cr} = P_{Cr} / P_d = f(B, \cos\varphi)$

The characteristic of the normalized reactive power of the inductance L in Fig. 1a $P'_{Lr} = P_{Lr} / P_d = f(B, \cos\varphi)$ is graphically displayed in Fig. 8.

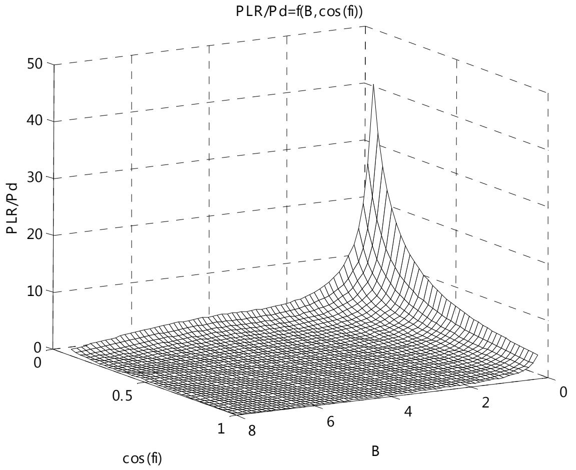

Fig. 8. The characteristic of the normalized reactive power of the inductance L $P'_{Lr} = P_{Lr} / P_d = f(B, \cos\varphi)$

V. Results Confirmation

Different approaches have been applied to confirm the validity of the calculated 3-D (surface) characteristics. They include conventional calculations, PSICE simulations, comparison with the already known 2-D (family) characteristics, etc.. All these means prove the correctness of the characteristics obtained.

Especially, the 3-D (surface) characteristics have been compared with the already known 2-D (family) characteristics (see [2], § 10 - 11, pp. 58 – 64, Fig. 10.1, Fig. 11.1 – Fig. 11.4). Even at a first glance it is obvious that the output, input and distortion 3-D and 2-D characteristics coincide.

For instance the 2-D family of turn – off (recovery) characteristics $TQC = f(B, \cos\varphi)$ is shown in Fig. 9. After comparing Fig. 6 with Fig. 9 it can be concluded that the sets of functions $TQC = f(B, \cos\varphi)$ are the same in both cases. Bit the 2-D families of characteristics are difficult to be applied practically while the 3-D (surface) characteristics are very detailed and accurate because they can be built with any desirable step and range of main parameters variations or even with different couples of main parameters.

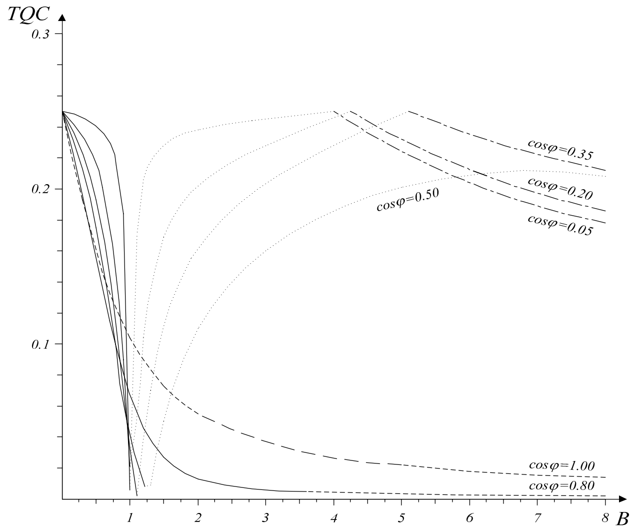

Fig. 9. The 2-D family of the turn – off (recovery) characteristics $TQC = f(B, \cos\varphi)$

VI. Conclusion

The 3-D characteristics summarize in a short but very convenient way the relationships between the parameters of an inverter whose power circuit could be reduced to the structures, shown in Fig. 1a or Fig. 1b and whose principle of control (turning on and off) of the semiconductor devices is a conventional one. Taking into account the capabilities of the contemporary MATLAB versions including zooming in and out, positioning the cursor on a desirable point on a given surface and reading the data assigned to it makes it possible to apply the characteristics for swift study and design of the power circuits, and for organizing an intelligent control of the inverters adjustable to load requirements and variations.

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